ASSESSMENT OF DYNAMICALLY LOADED CRACKS IN FILLETS

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Abstract
Practical methods for the safety assessment of postulated crack-like material defects of cubic containers made of ductile cast iron are presented. A formula for the stress intensity factor of a crack in a fillet with a radius from 20 to 200 mm under static load conditions is given. From that, an assessment diagram is derived for the critical depth of a crack in a fillet. The formula may be used to estimate the stress intensity factor of a dynamically loaded crack for special cases. As an application the results of the estimation procedure are compared with the results of a dynamic finite element calculation.

Introduction
BAM is the German competent authority for design assessment of packages for radioactive materials and develops in a research project improved fracture mechanical assessment methods for cracks in the highest stressed regions of cubic containers made of ductile cast iron. For that purpose postulated surface cracks in fillets are analysed. The containers with postulated material defects have to fulfil the requirements of German licensing procedures for cask designs for radioactive materials. So far, a container for final disposal of radioactive materials has to withstand a drop test from a height of 5 m onto a rock underground. Thus, the fracture mechanics assessment concept must be able to evaluate also dynamically loaded cracks. In general, a complex dynamic three-dimensional analysis of the cask structure including cracks is necessary by numerical computation methods.

From the literature many calculation formulae are known for statically loaded cracks in components. The application of these well-known solutions to dynamically loaded cracks is a priori not allowed, since the prerequisites of these formulae are no longer fulfilled. The load of the crack configuration is given usually far away from the crack tip. Under consideration of dynamic stresses in a cask one states that the crack tip experiences the influence of a variation in load only deferred because of the finite propagation speed of the stress waves in the component. However, it can be shown that in special cases formulae for static loads are appropriate also in the context of a dynamic fracture mechanics computation.

The goal of the investigations is the development of formulae and diagrams that can be handled simply for the assessment of crack-like material defects, which are treated as sharp cracks here. In the first step the stress in the component without cracks is computed for a given load as a function of the load parameter, geometrical dimensions and material properties. Then the crack tip parameter (e.g. stress intensity factor) of a crack inside the component is determined for the same boundary conditions. From these computation results a correlation between the stress and the crack tip parameter is finally deduced, which is valid
for the selected boundary conditions. This provides the information to justify allowable loads or allowable crack-like defects.

![Cross section of a cubic container (a) with a crack in a fillet and quarter segment for calculation (b).](image)

**FIGURE 1.** Cross section of a cubic container (a) with a crack in a fillet and quarter segment for calculation (b).

**Static formulae for a crack in a fillet**

A surface crack in a fillet is investigated. The real three-dimensional material defect is modelled conservatively by a two-dimensional crack at plane strain conditions. In typical load situations of the investigated cubic containers the tension stress in the container walls was comparatively small with approximately 10% of the bending stress. Therefore a bending stress $\sigma_b$ is supposed as a replacement load in the determination of the static fracture mechanics formulae. The geometry function $Y_{b,H}$ for the crack configuration is arranged as a function of conditions $B/W$, $a/W$ and $R/W$ ($B$ – container width, $W$ – container wall thickness, $a$ – crack depth, $R$ – fillet radius). Thus the following equation results for the stress intensity factor of a surface crack in a fillet:

$$K_{I,H} = Y_{b,H}(B/W, a/W, R/W)\sigma_b\sqrt{a}$$

Fig. 1 illustrates the bending condition in the quarter segment of the cubic container. In the centre of the wall a virtual referred transverse force $q$ works, which corresponds to the displacement $\Delta$ in the centre of this wall. From this, both the referred bending moment $m$ and the tension stress $p$ arise.

Fig. 2 shows the finite element mesh used for the calculation of the surface crack with infinite length and finite depth in a fillet with a radius of 50 mm. Similar models were created for the other investigated crack depths ($a = 2$ mm; 4 mm; 8 mm; 32 mm; 64 mm; 128 mm). The radius of the fillet is an additional influence quantity. Calculations for four fillet radii $R$ ($R = 20$ mm; 50 mm; 125 mm; 200 mm) were carried out with the commercial code ABAQUS to determine the crack tip parameter as a function of the radius of the fillet by a fitting procedure. For the reduction of the calculation effort the wall thickness is assumed to
be constant with a value of 160 mm which is the wall thickness of the German design of a radioactive waste container “Type VI”. For the container width three different values are investigated: \(B = 1450\) mm; 2000 mm; 1680 mm. They correspond to the length of the shortest edge, the longest edge and the mean average value of all edge lengths of the investigated container structure. A separate finite element mesh is required for every combination from wall thickness \(W\), container width \(B\), fillet radius \(R\) and crack depth \(a\).

![Finite element mesh](image)

**FIGURE 2.** Finite element mesh for static two-dimensional calculations with a crack in a fillet with details (crack depth: 16 mm, fillet radius: 50 mm).

From the possible combinations of the parameters a calculation scheme was prepared. Altogether 192 calculations were carried out based on 8 crack depths (\(a = 0\) mm; 2 mm; 4 mm; 8 mm; 16 mm; 32 mm; 64 mm; 128 mm) and 24 combinations of fillet radius and container width. Based on test results as load a specific force \(q\) of 3 kN/mm is prescribed in the centre of the wall. Symmetry boundary conditions were defined in the wall centre and at the diagonal of the fillet.

For the determination of a critical crack depth \(a_c\) as a function of the maximum principal stress without a crack in the fillet, eq. (1) is rearranged with \(a = a_c\) to

\[
K_{1,\text{Mat}} - \left( \frac{\sigma_{1,\text{max}}}{F_{0,\text{H}}} \right) Y_{b,H} (a_c / W, B / W, R / W) \sqrt{\pi \, a_c} = 0
\]

(2)

with the geometry function \(Y_{b,H}\) for a crack in a fillet. Eq. (2) can be solved numerically and provides the crack depth \(a_c\) dependent on the material quality (expressed by the fracture toughness \(K_{1,\text{Mat}}\)) and the maximum principal stress \(\sigma_{1,\text{max}}\) at the same position. It has to be
taken into account that the radius of the fillet must be sufficiently large, because for an infinitely small fillet radius the stress gets singular. Then, a representation of the critical crack depth over stress would be no longer possible. For fillet radii greater than 20 mm in practice this problem doesn’t appear, though. The load factor $F_{0,H}$, that represents the relation between $\sigma_k$ in the wall centre and $\sigma_{1,\text{max}}$ in the fillet, depends on the radius of the fillet and must be identified separately for any investigated geometry with a calculation without a crack ($a = 0$ mm). We restrict ourselves to linear elastic material behaviour (Young’s modulus $E = 162500$ MPa, Poisson’s ratio $= 0.29$). A more detailed explanation can be found in Zencker [1]. An extension to elastic plastic material behaviour is also discussed there.

The geometry function $Y_{b,H}$ of a crack in a fillet is expressed by the approach function

$$Y_{b,H}(x) = \sum_{i=1}^{5} b_i x^{i/10}$$

(3)

with the normalised crack depth $x = a / \sqrt{2W}$. The coefficients were calculated for fillets with radii $R$ of 20 mm, 50 mm and 125 mm for linear elastic material behaviour by means of a fitting procedure (Table 1).

Table 1. Coefficients of the geometry function and load factor $F_{0,H}$.

<table>
<thead>
<tr>
<th>$R$ / mm</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$F_{0,H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-56.6966</td>
<td>365.3780</td>
<td>-761.5320</td>
<td>656.9990</td>
<td>-203.0220</td>
<td>2.40655</td>
</tr>
<tr>
<td>50</td>
<td>-4.6216</td>
<td>38.8600</td>
<td>-53.3099</td>
<td>6.0879</td>
<td>14.2101</td>
<td>1.50456</td>
</tr>
<tr>
<td>125</td>
<td>17.2519</td>
<td>-86.7126</td>
<td>190.9590</td>
<td>-192.8020</td>
<td>72.3306</td>
<td>0.93315</td>
</tr>
</tbody>
</table>

The geometry function is valid for crack depths $a$ in the range from 2 mm to 64 mm at a wall thickness $W$ of 160 mm and a ratio $W / B \approx 0.1$. The quotient $F_{0,H}$ from $\sigma_{1,\text{max}}$ and $\sigma_b$ is valid for a container width in the range of 1450 mm to 2000 mm and depends on the fillet radius $R$ (Table 1).

With Eq. (2) the representation of the critical crack depth $a_c$ is received as a function of the fracture toughness and the stress for linear elastic material behaviour. Fig. 3 shows the assessment diagram for a fillet radius of 50 mm. It must be noted for the assessment of a material defect with this diagram, that a safety factor equal to 1 was taken into account till now. Such a safety factor must be defined in a construction application in a suitable way.

Generalisation of the geometry function

In the previous representation a separate geometry function is needed for every radius of the fillet. It is therefore desirable to represent the crack tip parameter or the geometry function
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resp. in closed form as a function of the fillet radius. We write the stress intensity factor in the form:

\[ K_I = \frac{\sigma_{1,\text{max}}}{F_{0,h}} \left( \frac{a}{\sqrt{2W}}, \frac{R}{\sqrt{2W}} \right) \sqrt{\pi a} \]  

(4)

Now, the quotient \( F_{0,h} \) is a function of the ratio \( R/\sqrt{2W} \) and is described by the approach

\[ F_{0,h} \left( \frac{R}{\sqrt{2W}} \right) = \sum_{i=1}^{4} c_i \left( \frac{R}{\sqrt{2W}} \right)^{-i/10}. \]  

(5)

The function \( Y_{h,I} \) is expressed by

\[ Y_{h,I} = R_w \cdot B \cdot A^T \]  

(6)

with

\[ R_w = \left[ 1, \left( \frac{R}{\sqrt{2W}} \right)^{0.2}, \left( \frac{R}{\sqrt{2W}} \right)^{0.4}, \left( \frac{R}{\sqrt{2W}} \right)^{0.6} \right] \]

and

FIGURE 3. Critical crack depth in a fillet with a radius of 50 mm as a function of the maximum principal stress and the fracture toughness of the material.
\[
A_{IP} = \left[ \left( \frac{a}{\sqrt{2W}} \right)^{0.1}, \left( \frac{a}{\sqrt{2W}} \right)^{0.2}, \left( \frac{a}{\sqrt{2W}} \right)^{0.3}, \left( \frac{a}{\sqrt{2W}} \right)^{0.4}, \left( \frac{a}{\sqrt{2W}} \right)^{0.5} \right].
\]

By a fitting procedure one gets the coefficients

\[
c_1 = 4.81012 \\
c_2 = 10.49840 \\
c_3 = -8.42982 \\
c_4 = 3.38024
\]

as well as

\[
B = \begin{bmatrix}
1709.88 & -8663.87 & 16395.1 & -13637.8 & 4236.56 \\
-6194.11 & 32938.6 & -64422.6 & 55144.6 & -17478.9 \\
7101.9 & -39520.7 & 79956.7 & -70239.5 & 22726.5 \\
-2570.93 & 15034.8 & -31498.3 & 28406.6 & -9381.84
\end{bmatrix}.
\]

The geometry function is represented in Fig. 4a as a function of the crack depth and the fillet radius. One recognises well that the sphere of influence of a fillet fades away fast due to the geometry, so that radii greater than 50 mm cause hardly any improvement in the component. With the help of the geometry function the stress intensity factor also can be illustrated in dependence of the crack depth and the fillet radius. The stress intensity factor is shown in Fig. 4b for a constant bending stress of 200 MPa in the wall centre.

Dynamically loaded cracks

In the previous section formulae are given for the calculation of the crack tip parameter in typical components of a container for static load conditions. It shall be checked now whether these formulae are suitable for the estimate of the crack tip parameter also for time-dependent load conditions. Previous studies considering a plate confirm this (Zencker [2]), provided that the load changes are adequately slow so that there are quasi-static conditions in the
component. These investigations are repeated more practically with two-dimensional container models here.

Fig. 5 shows the finite element model with a crack for the simulation of a container drop from a height of 5 m onto a concrete target. The mesh around the crack tip is illustrated in detail in addition. The directly calculated dynamic stress intensity factor is given over time in the accompanying diagram as the so-called dynamic solution. The curve called "dynamic stress & BAM static solution" was found out formally from the above given formula for a statically loaded crack in a fillet with time-dependent stress. This stress results from a calculation without a crack. It was determined in the shortest distance to the imaginary position of the crack tip at the component surface. A good agreement of both curves can be seen. However, it is an important prerequisite for such a good coincidence that the crack depth is sufficiently small (1/10 of the wall thickness in the investigated case). Otherwise the crack influences the global stress field in the component and the curves then differ increasingly from each other. This procedure is limited also through the duration of the load or, what was shown already in another place, the rise time of the load (Zencker [3]).

The advantage of the introduced procedure is, that for the short load time during a typical container test the expensive dynamic calculation of a finite element model with a crack can be replaced by an approximation without a crack using our static formula. Nevertheless, a sufficiently small change of the load and a sufficiently small depth of the crack are necessary for reliable results. So the application of this methodology has to be verified for other specific applications different to the cases investigated here.

Conclusions

A formula for the stress intensity factor of a crack in a fillet with a radius from 20 to 200 mm under static load conditions was derived. The critical crack depth was calculated as a function of the stress, the material quality (given by the fracture toughness) and the wall thickness of the container for surface cracks in fillets. The application of static solutions of
crack problems to time-dependent load scenarios is possible in special cases if the crack depth is small (less than one-tenth of wall thickness) and the rise time of the load stress is “long enough” (typical time scale in milliseconds), i.e. if the crack tip behaviour can be considered quasi-statically in spite of a non-static load. Hence, the prerequisites of the static solutions used are not violated in our application. Any use of static equations with time-dependent parameters must be tested intensively in each case.

References


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