Comments on "Jump-like crack growth models or theory of critical distances. Are they correct?" by A Neimitz, ESISNewsletter #44, 20-26, June 2008

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Abstract
Aim of the present note is to correct an oversight in [1] that forced the Author to conclude that in linear plane elasticity problems jump-like crack models are derived and work only if the first term in the stress field asymptotic expansion is taken into account. The number of the equations as well as the symbols refer to the ones used in [1].

Remark
In order to prove that jump-like crack growth models have a limited validity, in [1] the Author took into account also the second term in the asymptotic stress field at crack tip (eqn 17) and substituted it into Novozhilov failure criterion (eqn 10), which is probably the originator of jump-like crack growth models. Physically meaningless results were found (e.g. the tensile strength is not a material property – see fig.7). The Author concluded that jump-like models work only if Griffith geometry and the first term in the stress asymptotic expansion is considered.

However, an oversight was committed in the proof: the stress field (eqn 17, given in a reference system with origin at the crack tip) has to be integrated over the interval $[0, \Delta a]$ and not over $[a, a + \Delta a]$. Hence, eqns (18), (19), (20) (as well as fig. 7) are wrong and the proof does not hold.

On the other hand, by using the correct extremes of integration (and assuming that $\sigma_m=\sigma_u$ in eqn(10) as in [2]), together with eqn (1), the following equation is obtained:

$$K_I + A_I \, \Delta a = K_{Ic} \tag{a}$$

Eqn (a) is meaningful: it highlights the (first) corrective term with respect to LEFM, which is recovered only for extremely brittle materials ($\Delta a \to 0$).

Conclusions
Jump-like models (such as [2-3]) are formally correct. The debatable point is if they describe real material behaviour. Obviously, they are expected to work for brittle or quasi-brittle materials, since the basic equations hold for linear elasticity. Any other extension should be carefully considered, as done by Neimitz himself in [4] for elastic-plastic materials.

References
[1] A. Neimitz (2008). *Jump-like crack growth models or theory of critical distances. Are they correct?* ESISNewsletter #44, 20-26, June 2008 and Proceedings of ECF17, Brno, Czech Republic, 2-5 September, 2008.
[2] P. Cornetti, N. Pugno, A. Carpinteri, D. Taylor (2006). *Finite fracture mechanics: a coupled stress and energy failure criterion*. Engineering Fracture Mechanics. 73, 2021–33.
[3] D. Leguillon (2002). *Strength or toughness? A criterion for crack onset at a notch*. European Journal of Mechanics A/Solids, 21, 61–72.
[4] A. Neimitz (2008). *The jump-like crack growth model, the estimation of fracture energy and J_R curve*. Engineering Fracture Mechanics, 75, 236-252.