

**TC16 – Finite Fracture Mechanics  
Workshop I – 13-14 April,  
Hotel Boton D'or, La Thuile, Italy  
Program and list of Abstracts**

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**Friday, Apr 13**

**13:00 – 13:30** Opening – Zohar Yosibash and Dominique Leguillon

**13:30 – 14:15** Finite Fracture Mechanics: stress weight functions, friction, stability and symmetry

Pietro Cornetti and Alberto Sapora

Department of Structural, Building and Geotechnical Engineering, Politecnico di Torino, Torino, Italy

**14:15 – 15:00** 3D applications of the coupled criterion

Aurélien Doitrand\* and D. Leguillon\*\*

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**15:00 – 15:45** Coffee break

**15:45 – 16:30** The intrinsic tensile strength of a ceramic

D. Leguillon

Institut Jean Le Rond d'Alembert, Sorbonne université, CNRS, 4 place Jussieu, 75005 Paris, France

**16:30 – 17:15** Analysis of cracks at elastic interfaces by Finite Fracture Mechanics

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**19:00** Dinner

## **Saturday, Apr 14**

**9:30 – 10:15** A Discrete Damage Model for Woven Ceramic Matrix Composites

E. Martin

Laboratoire des Composites Thermo-Structuraux, Université de Bordeaux, CNRS UMR 5801, F-33600, Pessac, France.

**10:15 – 11:00** On the upper bound of finite crack lengths in mixed mode fracture problems

P.L. Rosendahl and W. Becker

Technische Universität Darmstadt, Fachgebiet Strukturmechanik, Franziska-Braun-Str. 7, 64287 Darmstadt, Germany

**11:00 – 11:30** Coffee break

**11:30 – 12:15** Micro-cracks, scratching and fragmentation in annealed and thermally tempered glass

Jens Schneider, Steffen Müller-Braun and Navid Pourmoghaddam

Technische Universität Darmstadt, Institute of Structural Mechanics and Design, Franziska-Braun-Str. 3, 64287 Darmstadt, Germany

**12:15 – 13:15** Lunch break

**13:15 – 14:00** Mode-I and mixed-mode specimens for evaluating failure models of brittle failure

Philipp Weißgraeber\* and Julian Felger \*\*

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**14:00 – 15:15** Round table – Coordinator Zohar Yosibash

1. World-wide FFM exercise – assignment of a series of experiments and theoretical approaches to predict fracture load and failure initiation.
2. Executive committee of TC16
3. Second workshop – date, place, organizer
4. Possible collaborative publications and research.

**15:15** Adjournment

## **Finite Fracture Mechanics: stress weight functions, friction, stability and symmetry**

**Pietro Cornetti and Alberto Sapora**

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The presentation provides miscellanea of topics, all related to FFM, currently under investigation at the Politecnico di Torino, in cooperation with other European research groups. The first topic explores the similarities between the Cohesive Crack Model and the Finite Fracture Mechanics: we show that an excellent matching between the two models can be found by introducing weight functions in the stress requirement within the Finite Fracture Mechanics approach. A correspondence rule between weight functions and cohesive laws is provided. The second topic aims at extending the Finite Fracture Mechanics approach for interfacial debonding to take friction (in its simplest form i.e. a constant residual stress) into account. We show that semi-analytical solutions can be found, e.g., for the pull-push or the pull-out tests. The third topic deals with negative geometries faced by Finite Fracture Mechanics: particularly, we consider a holed plate under biaxial loading: varying the biaxiality ratio, it is possible to span (almost) all the possible structural behaviors: completely positive, locally negative but globally positive, locally positive but globally negative geometries. Correspondingly, we investigate the stability of crack growth. The fourth and last topic is symmetric vs. asymmetric crack propagation: it is shown that, differently from LEFM, Finite Fracture Mechanics provides different failure loads depending on the kind of crack propagation. Applications are provided for the Griffith crack and the double lap joint geometries.

## 3D applications of the coupled criterion

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The coupled criterion (CC), proposed by Leguillon [3], is an efficient method for crack initiation prediction in brittle materials. It is based on the fact that crack nucleation requires both a stress and an energy conditions to be fulfilled simultaneously. The CC has been widely used to study crack initiation (cf. the extensive review by Weissgraeber et al. [5]) for various materials (composites, ceramics, rocks, etc) under several kinds of loadings. It works also well for interfaces and joints. Until now, the coupled criterion has mainly been used in 2D applications. It is possible to extend it to some 3D cases, since its formulation holds valid in 3D [4]. However, contrary to 2D cases, where the crack is generally described by only one or two parameters (its length and possibly its direction if unknown), the main difficulty of the 3D case is that the crack may be described by an infinite set of parameters [1, 5]. To the authors knowledge, there are only few 3D applications of the coupled criterion in the literature [1, 2, 4]. This work addresses 3D applications of the coupled criterion to crack initiation load and shape determination in the case of :

- o Four point bending of aluminum/epoxy bimaterial specimens
- o Tensile loading and bending of scarf adhesive joints
- o Crack front segmentation into facets under mode I+III loading

### References

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## The intrinsic tensile strength of a ceramic

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Criteria for predicting initiation of cracks in brittle materials like ceramics (Finite Fracture Mechanics, Cohesive Zone Models, Phase Field methods) are based on two parameters that can always be reduced to the material fracture toughness and the tensile strength. Standardized experiments exist to estimate the former, whereas measuring the latter is more challenging. The reported strength is commonly strongly scattered due to the critical defect size distribution on the tested specimens. The statistical distribution is represented by the well-known Weibull law. In this work, we propose a definition of the “intrinsic” tensile strength to be used in numerical models, making a distinction between extrinsic defects due to manufacturing and intrinsic ones relying on the microstructure (e.g. grain size). Unlike the measured strength, this intrinsic strength is a deterministic parameter, it depends in particular on the grain size of the material.

Our approach is based on the Finite Fracture Mechanics theory and the Coupled Criterion (CC) [1-2]. The CC is applied to small surface flaws and the extrinsic tensile strength is exhibited as a function of the flaw depth. Numerical results show a good agreement with experiments on alumina reported in the literature [3]. Using the intrinsic tensile strength, the CC can predict crack initiation both from flaws and from larger geometric perturbations like V- or U- notches made for experimental purposes.

In addition, a model for Petch's law (strength vs. grain size) [4] in polycrystalline materials is proposed using the CC. It is shown that the initial crack length includes an increasing number of grains as the grain size decreases. Moreover, the residual thermal stresses due to the anisotropy of the grains and their random distribution can play a role to explain some features of the Petch law.

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## **Analysis of cracks at elastic interfaces by Finite Fracture Mechanics**

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The integrity of composite structures is determined by the strength and durability of their adhesive joints and interfaces in general, among other conditions. Thus, one of the main concerns in the design composite structures is to characterize adequately interfaces between adjacent solids, to compute stress distributions at these interfaces and to apply a suitable failure criterion therein. A great diversity of joint and interface models is available in the literature [1,2]. To find a suitable model for each case is still a difficult task. Each model tries to represent the real behavior of an interface, capturing its main features. The simple and double lap joint (SLJ and DLJ) tests are considered the most used tests for interfaces, with the pull push test (PP) being a modified version of DLJ.

In this work, the behaviour of these joints/interfaces is modelled by a distribution of linear elastic springs which represents the adhesive, see Lenci [3] for a review. Apparently, Volkersen [4] and Goland with Reissner [5] were the first who used this model. Since then many related works have been published, nevertheless in some cases the predictions do not coincide with the experimental evidence, that is why the model is still studied in order to find a way to improve it.

Cornetti et al. [6] presented a study of PP test using the coupled criterion of FFM applied to a 2D linear elastic interface, where the shear stress distribution was only considered. The present work continues this study of the PP test but adding the effect of the peeling stress. Additionally, the DLJ test is also studied. Eventually, the analytical models used here are compared with a numerical study based on the same criterion and implemented in a BEM code, which was satisfactorily tested for the debond onset at the fiber-matrix interface in a unidirectional composite laminate [7].

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## **A Discrete Damage Model for Woven Ceramic Matrix Composites**

**E. Martin**

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Ceramic matrix composites made with a silicon carbide matrix reinforced with woven silicon carbide fibers demonstrate low density and high strength. These materials can be used at high temperatures up to 1250°C and are good candidates to replace superalloys in gas turbine engines in aircrafts. The mechanical behavior of SiC/SiC composites exhibits a non-linear behavior which results from the progressive development of various crack networks. A finite fracture approach based on the Coupled Criterion is here used to analyze the cracking mechanisms at the mesoscale. In-situ micrographic observations obtained during tensile and bending tests are utilized to identify three families of cracks which include matrix cracking, transverse tow cracking and longitudinal tow cracking. A two dimensional finite element model is generated to approximate the actual specimen section geometry including matrix, fiber tows and porosity. Numerical simulations are carried out with a dedicated algorithm to simulate nucleation and propagation of cracks [1]. Comparing the simulation results with experimental ones shows that the model captures the main cracking features [2]. The proposed approach is a simple and robust method to predict discrete damage in ceramic matrix composites.

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## On the upper bound of finite crack lengths in mixed mode fracture problems

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Many crack nucleation problems for which finite fracture mechanics provides accurate failure load predictions involve cracks forced on a specific path. Such situations typically impose mixed mode conditions. Among others, this can concern crack initiation in composite laminates, delamination, or crack onset along interfaces such as in adhesive bonds.

While many adhesive joints employ linear-elastic adhesives, bonded connections may also rely on hyperelastic polymers whose molecular structure engenders remarkable deformability. Elastomers are amorphous cross-linked polymers which exist above their glass transition temperature. This allows for considerable segmental motion and typically yields hyperelastic behavior, high failure strain and low Young's modulus at ambient temperatures.

In finite fracture mechanics studies of multiaxially loaded linear-elastic materials, fairly simple stress criteria for the upper bound of the finite crack length usually suffice. Such criteria are for instance maximum principal stress or a quadratic interaction of circumferential and tangential stress.

Elastomers behave fundamentally different. Using different simple stress criteria developed for metals, early studies and own work on the fracture of unnotched multiaxially loaded rubber-like polymers [1, 2] were unable to represent test data satisfactorily. Owing to large deformations of hyperelastic materials prior to failure, tracking of cross sectional changes for the computation of true failure stresses is particularly challenging. Thus, recent analyses [3, 4] employ principal stretches to characterize mixed mode failure of elastomers.

Using hyperelastic polymers as an example, the present work reviews modern generalizations of classical failure criteria with respect to their suitability as an upper bound for the crack length in finite fracture mechanics. It is shown that the generalized Podgórski-Bigoni-Piccolroaz criterion [5, 6] allows for an accurate description of multiaxial failure of various hyperelastic elastomers such as natural and synthetic rubber, filled and unfilled vulcanizates as well as silicones in principal stretch space. The methodology can be readily transferred to principal stresses and other materials.

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## Micro-cracks, scratching and fragmentation in annealed and thermally tempered glass

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Glass is often used as a reference for an almost ideal brittle and isotropic material. The theoretical tensile strength of soda-lime-silica glass is more than 10 GPa, but the fracture toughness is well below  $1 \text{ MPa m}^{1/2}$  and thus microcracks, typically on its surface resulting from production, handling, transport, cleaning, or in the bulk resulting from internal imperfections like bubbles or inclusions [1] govern its strength for engineering design. Moreover, subcritical cracks growth influenced by humidity at the crack tip influences the crack propagation well below its fracture toughness [2].

Although LEFM can be well used to predict the macroscopic glass strength based on assumptions for the crack geometry and crack depth, engineering methods today use Weibull-statistics to deduce design values for practical glass design as it is still not clear how exactly the micro-cracks are initiated and which geometry and depth results from external actions and contact to other materials [3, 7]. On the other hand, the final fragmentation of the glass and its crack pattern after crack propagation, which strongly depends on the elastic energy resulting from external loads and residual thermal stresses induced during production (e.g. glass tempering, [4]), is important for the prediction of the post-fracture behaviour of the glass, e.g. in laminated glasses where glass is combined with transparent polymer interlayers [5].

We want to show experimental results for annealed and tempered glass of (i) micro-cracks and their development resulting from local indentation, scratching [6, 7] or glass cutting, (ii) the crack development and fragmentation of glass as a function of residual thermal stresses [8] and (iii) the crack propagation at the interface between glass and polymer interlayer during delamination [9]. In this context, we want to show the current limitations of LEFM and cohesive zone models to discuss to what extent finite fracture mechanics and the coupled criterion could be used improve glass failure prediction.

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## **Mode-I and mixed-mode specimens for evaluating failure models of brittle failure**

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A comprehensive set of different specimens that cover non-singular stress concentrations at elliptically shaped notches as well as singular stress raisers in form of sharp V-notches, is used to evaluate different failure models of brittle failure. In total 10 different geometries are studied.

The flat specimens are manufactured by hot curing 1-K epoxy in a custom casting form followed by water jet cutting of the individual geometries. The dimensional accuracy is measured by microscope analysis on three specimens of each type. The actual dimensions are given in detail. Additionally, the surfaces of the front and back side of the specimens as well as the surface of the water jet cut have been analyzed by scanning electron microscopy. The specimens are then tested in a universal testing machine under tensile loading with a sample size of eight for each geometry. The specimens are designed to study the influence of the ratio of the half axes of elliptical notches as well as the opening angle of V-notches on the effective strength under mode-I and mixed-mode loading conditions.

For comparison the coupled stress and energy criterion is evaluated numerically using finite element analyses. Additionally, a semi-analytic asymptotic approach is employed assuming that the length of initiating crack is small compared to a characteristic structural length. The semi-analytic asymptotic approach allows for an efficient study of the unknown crack angles in case of the mixed-mode specimens. The results could also be used for thorough comparison between different approaches to model crack initiation in brittle fracture, such as strain energy density methods or phase-fields models. In order to allow for such comparisons in a credible manner, in the present work the failure loads are only given as dimensionless and relative values.