K.-H. Schwalbe, A. Cornec*, and J. Heerens**

The Engineering Treatment Model (ETM) – a Simple Method for Estimating the Driving Force under Elastic–Plastic and Plane Stress Conditions

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ABSTRACT The Engineering Treatment Model (ETM) consists of a set of simple equations which serve for estimating the J integral or the crack tip opening displacement as driving force parameters in small scale yielding as well as in fully plastic conditions under the assumption of plane stress and of a piece-wise power law behaviour of the material's stress strain curve. It is shown that the applied load, applied strain, J integral, and crack tip opening displacement are interrelated through simple expressions. The ETM predictions are validated by means of experimental results and finite element calculations, both on laboratory specimen configurations.

Notation

a	Crack length
a_0	Fatigue pre-crack length
a_{eff}	Plasticity corrected crack length
E	Young's modulus
F	Applied load
G	Linear elastic strain energy release rate
G_{pl}	Plasticity corrected G
J	J integral
K	Linear elastic stress intensity factor
K_{eff}	Plasticity corrected K for tension
K_{pl}	Plasticity corrected K, general
$\bar{K}$	Plasticity corrected K for bending $= 0.5(K_{eff} + K)$
n	Strain hardening exponent
s	Load line displacement
s_c	s due to crack
s_{nc}	s without crack
W	Characteristic width of specimen or structure
ε	Applied strain
δ	Crack tip opening displacement

* GKSS-Forschungszentrum Geesthacht GmbH, D-2054 Geesthacht, FRG.

δ_5 Operational definition for experimental determination of δ; subscript 5 indicates gauge length of 5 mm
σ Applied stress
Subscript Y refers to value at yield load, F_Y.

Introduction

The assessment of the severity of a crack in a structural part requires the knowledge of the two quantities:

- driving force as a measure of the stresses and strains in the vicinity of the crack tip, expressed in the parameters stress intensity factor, K, J integral, or crack tip opening displacement (CTOD) with the symbol δ;
- the material's resistance against crack growth, either as a single value parameter (the fracture toughness) or as the crack growth resistance curve (R curve), both expressed in terms of K, J, or δ.

The determination of the latter quantity is well documented in various test standards or documents which are supposed to become standards eg **(1)–(4)**.

As far as the driving force is concerned, the regime of linear elastic fracture mechanics does not present major problems. The stress intensity factor can be expressed in the form

$$K = \sigma\sqrt{(\pi a)}Y(a/W) \tag{1}$$

where $Y(a/W)$ is a calibration function which depends on geometrical factors only (geometry of the part under consideration, of the loading, and of the crack). Values for the calibration function for many configurations can be found in the literature **(5)(6)**. On the other hand, under elastic–plastic and fully plastic conditions the situation is much less convenient. The driving force depends in a complicated manner on both the geometry and the material's stress–strain curve.

Although the quantities desired, like the driving force J or δ, or displacements like the load line displacement, can be adequately determined by numerical methods, it is necessary to have analytical relationships even if they are based on simplifying assumptions. The benefit of such analytical relationships is obvious, since a formula shows clearly the effect of the relevant parameters on the quantity desired and they can serve for quick assessments of structural problems. This is why assessment methods like the COD design curve with its recent modifications **(7)(8)** and the R6 failure assessment diagram **(9)** have been established in the past. They are well documented in the cited references and shall not be described in detail in the present paper. Their wide spread use shows clearly the necessity of procedures of this kind.

Recently, the engineering treatment model (ETM) was introduced which is mainly based on the CTOD, but it can also be applied to the load line displacement and the J integral **(10)–(12)**. Starting from a set of simplifying assumptions, simple analytical expressions have been derived which are intended to predict CTOD or the J integral as driving force parameters, without built-in safety factors. It should be mentioned here that within the framework of ETM the CTOD refers to the experimental definition δ_5 which measures the CTOD at the specimen's side surface, spanning the original fatigue crack tip over a gauge length of 5 mm **(13)(14)**. This definition is numerically close to values determined after BS 5762.

In the present paper a brief overview on the main items of ETM will be given. It will then be shown that δ_5 as predicted by ETM can be easily converted to the J integral; with this, direct conversions exist between the stress intensity factor, K, the J integral, and the CTOD. Extensive validation of ETM has been performed so far on laboratory specimens tested at GKSS and on some finite element calculations carried out at the Technische Hochschule Darmstadt. Some examples of these will be shown in the paper. Furthermore, ETM will be compared with the EPRI Handbook and with Turner's J design curve formulation. The paper then concludes with failure prediction.

Basic ETM formulae

Assumptions

The cracked part is assumed to deform in a state of prevailing plane stress which means that the predictions can be expected to be more accurate for relatively thin cross sections than for thick sections.

It is anticipated, that the material's stress–strain curve can be approximated by the piece-wise power law

$$\frac{\varepsilon}{\varepsilon_Y} = \left[\frac{\sigma}{\sigma_Y}\right]^{1/n}; \quad \sigma > \sigma_Y \tag{2}$$

For convenience, the yield strength, σ_Y, is set equal to the proof stress, $\sigma_{0.2}$.

Crack tip opening displacement

Below the yield load, F_Y, which is the load at which the net section attains the yield condition, the CTOD in terms of δ_5 is supposed to be given by the small scale yielding, non-hardening solution

$$\delta_5 = \frac{K_{pl}^2}{E\sigma_Y} \tag{3}$$

where K_{pl} is the plasticity corrected stress intensity factor, which can be approximated for tension configurations by

$$K_{pl} \equiv K_{eff} = \sigma\sqrt{(\pi a_{eff})}Y(a_{eff}/W) \tag{4}$$

with

$$a_{\mathrm{eff}} = a + \frac{K^2}{(1+n)2\sigma_{\mathrm{Y}}\pi} \tag{5}$$

and Y being the calibration function for the geometry under consideration. Bending configurations can be characterised by

$$K_{\rho 1} \equiv \bar{K} = 0.5(K_{\mathrm{eff}} + K) \tag{6}$$

If the loading configuration cannot be clearly identified as tension or bending, equation (4) is to be preferred.

Figure 1 shows predicted versus measured values of δ_5. Equation (3) yields predictions with reasonable accuracy up to the yield load, F_{Y}. It must be mentioned, however, that below $0.5F_{\mathrm{Y}}$ equation (3) yields improper results as compared with experimental δ_5 determinations. At loads greater than F_{Y} the net section deforms proportional to $(F/F_{\mathrm{Y}})^{1/n}$ **(12)**. The behaviour of the yielding net section may be characterised by any suitable displacement along a gauge length spanning the cracked cross section, in particular by δ_5. Thus, recalling equation (2), strains or displacements are given by their respective

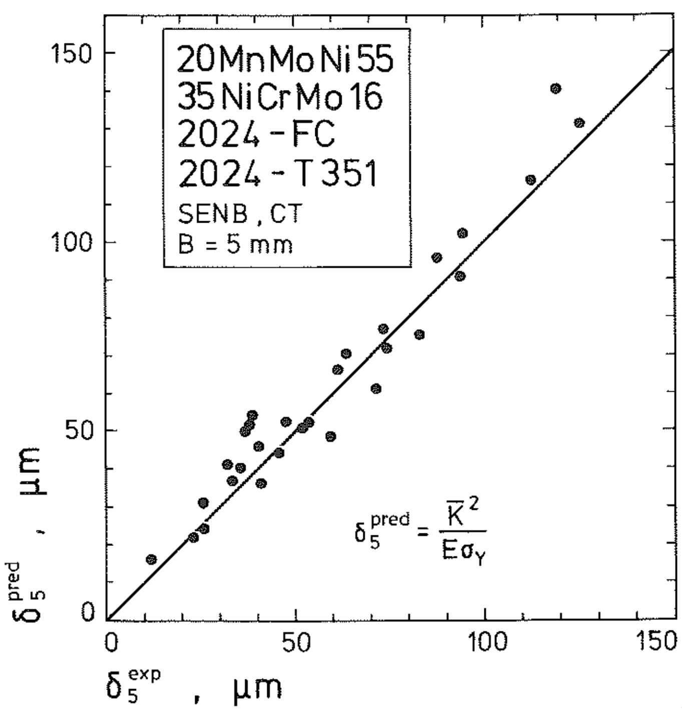

Fig 1 Experimental and predicted CTOD in terms of δ_5 for SENB and CT specimens made of four materials; loading conditions: $F = (0.4 \ldots 1)\, F_{\mathrm{Y}}$ (12)

values at yield load times $(F/F_{\mathrm{Y}})^{1/n}$, in particular

$$\frac{\delta_5}{\delta_{\mathrm{Y}}} = \left(\frac{F}{F_{\mathrm{Y}}}\right)^{1/n} \tag{7}$$

with δ_{Y} being the value of δ_5 as predicted by the small scale yielding solution according to equation (3) at $F = F_{\mathrm{Y}}$. Equation (7) is a geometry independent driving force expression which depends on the material through the hardening exponent, n. The example shown in Fig. 2 shows good coincidence between experiment and prediction.

Figure 3 shows schematically how δ_5 behaves as a function of the applied load along with the estimation formulae equations (3) and (7).

An alternative formulation for δ_5 under fully yielding conditions and for $a/W \to 0$ expresses δ_5 as a function of the applied strain

$$\frac{\delta_5}{\delta_{\mathrm{Y}}} = \frac{\varepsilon}{\varepsilon_{\mathrm{Y}}} \tag{8}$$

which is a geometry and material independent driving force formulation.

For the infinitely wide plate under tension $(a/W \to 0)$ the stress intensity factor is given by $K = \sigma\sqrt{(\pi a)}$, δ_{Y} is calculated with $\sigma \to \sigma_{\mathrm{Y}}$, and with the plasticity correction of equation (5) one obtains the simple relationship

$$\delta_5 = 1.5\pi a\varepsilon \tag{9}$$

Load line displacement

The load line displacement, s, can be partitioned into a crack contribution, s_{c}, and a no-crack contribution, s_{nc}. Under the conditions of contained yield $(F \leqslant F_{\mathrm{Y}})$ the linear elastic solutions with a_{eff} instead of the actual crack length,

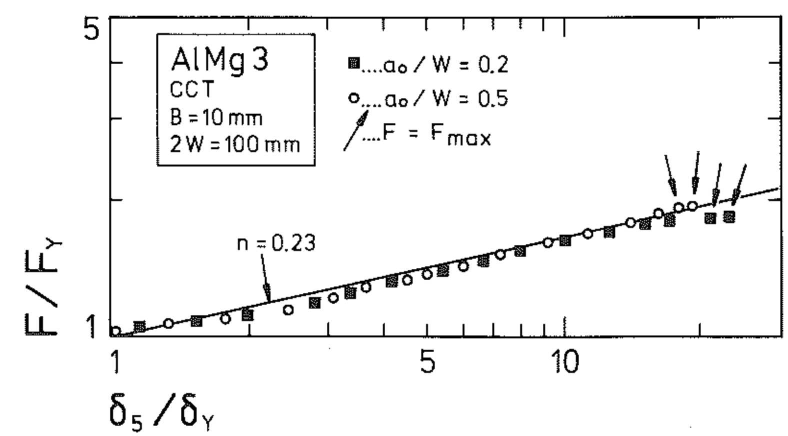

Fig 2 Crack tip opening displacement, δ_5, as a function of the applied load for four CCT specimens of the aluminum alloy AlMg3 (16). The n value shown in the graph is the strain hardening exponent determined in the tensile test

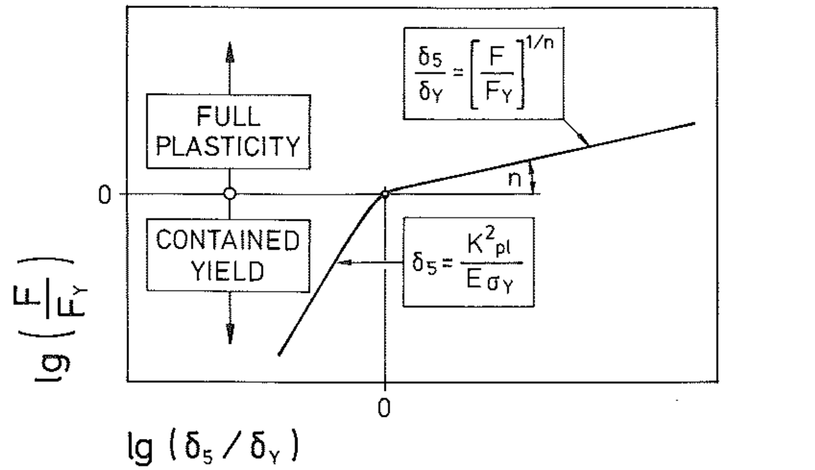

Fig 3 Schematic showing estimation formulae for δ_5 in relation to the loading ranges contained yield and full plasticity

a, gives quite accurate predictions. In the net section yielding case, s_{nc} remains elastic, and s_c can be estimated according to the way shown for δ_5

$$s = s_{nc} + s_{cY}\left(\frac{F}{F_Y}\right)^{1/n} \tag{10}$$

where s_{cY} is the value for s_c at $F = F_Y$. Figure 4 compares the load line displacement measured on a CCT specimen of the steel 35NiCrMo16 with the ETM prediction. In this experiment the elastic compliance of the starting

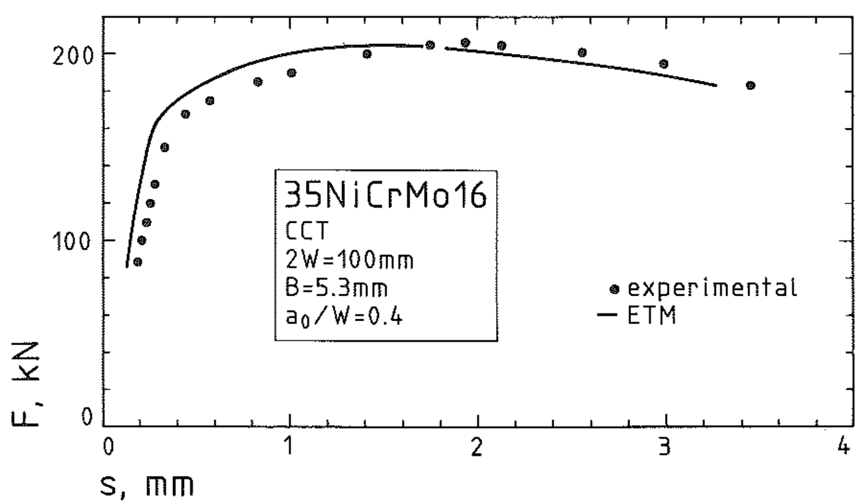

Fig 4 Experimental load line displacement, *s*, compared with ETM prediction

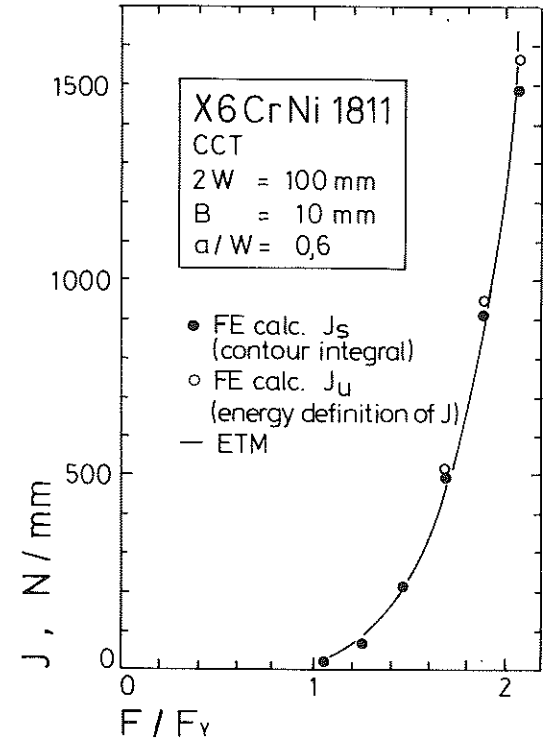

Fig 5 *J* integral for a CCT specimen of the austenitic steel X6CrNi1811, plane stress finite element calculations by Amstutz and Seeger (18) and ETM prediction

crack length, a_0, was about 25 percent higher than the theoretical one which explains the deviations. Thus, if an effective modulus of elasticity from the experiment had been taken, an almost perfect prediction had resulted. Further examples are given in reference (12).

J integral

Under contained yielding conditions J is simply given by

$$J = G_{pl} = \frac{K_{pl}^2}{E} \tag{11}$$

with K_{pl} according to equations (4) and (6). In the fully plastic regime **(17)**

$$J = J_{el} + J_{pl} \tag{12a}$$

$$= G + \eta_{pl}\frac{U_{pl}}{B(W-a)} \tag{12b}$$

with U_{pl} being the plastic work done on the specimen.

For a rigid–plastic material, $U_{pl} = s \cdot F_Y$. It is assumed that for an elastic–work hardening material the plastic work can be set equal to the plastic portion, s_{pl}, of the load line displacement times an average flow load, F_F, which is equal to the average of the actual load and the yield load, F_Y. Hence

$$J = G + \frac{\eta_{pl} s_{pl}}{B(W - a)} \frac{F + F_Y}{2} \tag{13}$$

If general yielding (i.e., gross section yielding) occurs, only that part of s_{pl} has to be taken which is due to the crack.

In Fig. 5 a prediction using equation (13) is compared with finite element calculations for a CCT specimen of an austenitic steel. The load line displacement necessary for estimating J was calculated using equation (10) along with the linear elastic solution for s of a CCT specimen provided by reference **(6)**. It can be seen that ETM gives a very accurate estimate of J.

Alternative J formulations

Turner introduced a J design curve for which he proposed several equations **(17)**, one of which being

$$\frac{J}{G_Y} = 2.5\left(\frac{\varepsilon}{\varepsilon_Y} - 0.2\right) \quad \text{for } \frac{\varepsilon}{\varepsilon_Y} \geqslant 1.2 \tag{14}$$

This expression suggests geometry and material independence which is due to the normalisation of J by G_Y.

According to the EPRI Handbook **(15)** the J integral for a piece-wise power law hardening material can be formulated using a plasticity corrected linear elastic term and a fully plastic contribution from a pure power law (PPL) hardening material, i.e.

$$J = G_{pl}; \quad \text{for } F \leqslant F_Y \tag{15a}$$

$$J = (G^*_{pl} - J^*_{PPL}) + J_{PPL}; \quad \text{for } F \geqslant F_Y \tag{15b}$$

$$J_{PPL} = J^*_{PPL}\left[\frac{F}{F_Y}\right]^{(1/n)/n} \tag{15c}$$

where G_{pl} is the plasticity corrected strain energy release rate, G, at $F = F_Y$ and J^*_{PPL} is a geometry dependent fully plastic term which has been determined by finite element calculations for a few geometries. J^*_{PPL} in turn is given by

$$J^*_{PPL} = \sigma_Y \varepsilon_Y a \hat{J}\left(\frac{a}{W}, n\right) \tag{16}$$

where $\hat{J}(a/W, n)$ is given in **(15)** in the form of tabulated values.

Introducing $G_Y = \sigma_Y^2 \pi a/E$ (which is, of course, valid for the finite body only) leads to

$$\frac{J_{PPL}}{G_Y} = \frac{1}{\pi} \hat{J}\left(\frac{a}{W}, n\right)\left[\frac{F}{F_Y}\right]^{(1+n)/n} \tag{17}$$

The structure of these J expressions suggests an alternative formulation which is compatible with the ETM formalisms (see, e.g., equations (7) and (10))

$$\frac{J}{J_Y} = \left(\frac{F}{F_Y}\right)^{(1+n)/n}; \quad \text{for } F > F_Y \tag{18}$$

Here J_Y is the value of J at $F = F_Y$, whereas G_Y in equations (14) and (17) is the linear elastic part of J at $F = F_Y$.

J_Y is simply calculated using the conversion

$$J = \frac{K_{pl}^2}{E} \tag{19}$$

with K_{pl} from equations (4) or (6) for $F = F_Y$. Thus, the prediction of J in the fully plastic regime requires just the linear elastic solution for K, the limit load solution (F_Y) and the strain hardening exponent.

In Fig. 6 J values for a CT and a CCT specimen obtained by plane stress finite element calculations are plotted on double logarithmic scales. The slope $n/(n + 1)$ of the straight line has been calculated using that value of n which was determined in the tensile test as the average slope of a double logarithmic σ–ε plot. Thus, the straight line in Fig. 6 represents a prediction and it is not drawn as the average line through the data points. It is obvious that the relationship between J/J_Y and F/F_Y is the same for both specimen types (which may represent some extreme loading cases, i.e., tension and predominantly bending), and that equation (18) represents a reasonable prediction formula for J as a driving force parameter. Figure 7 presents absolute J values

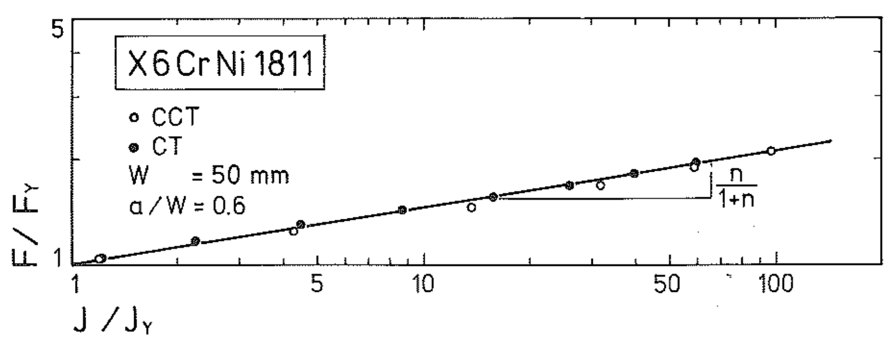

Fig 6 J/J_Y **plotted as a function of** F/F_Y **for an austenitic steel, plane stress finite element calculations by Amstutz and Seeger (18)**

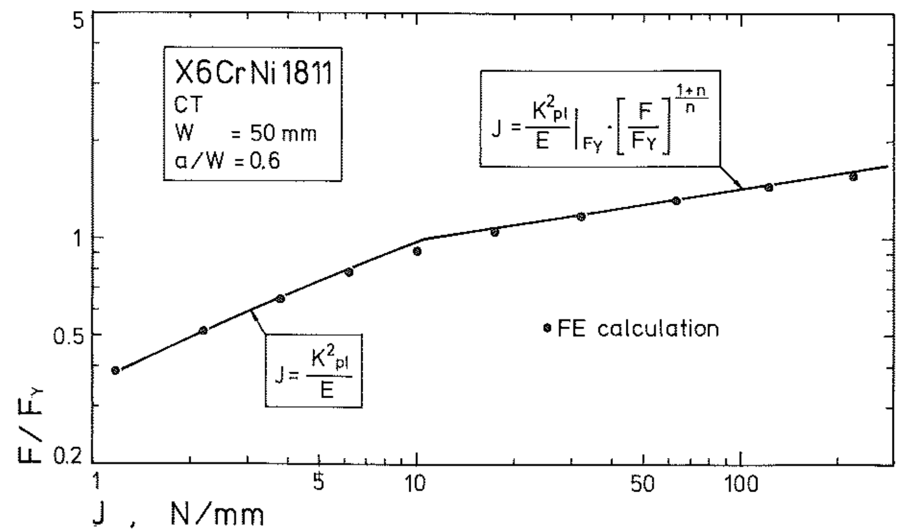

Fig 7 *J* values of a CT specimen determined by finite element calculations (18) and by ETM

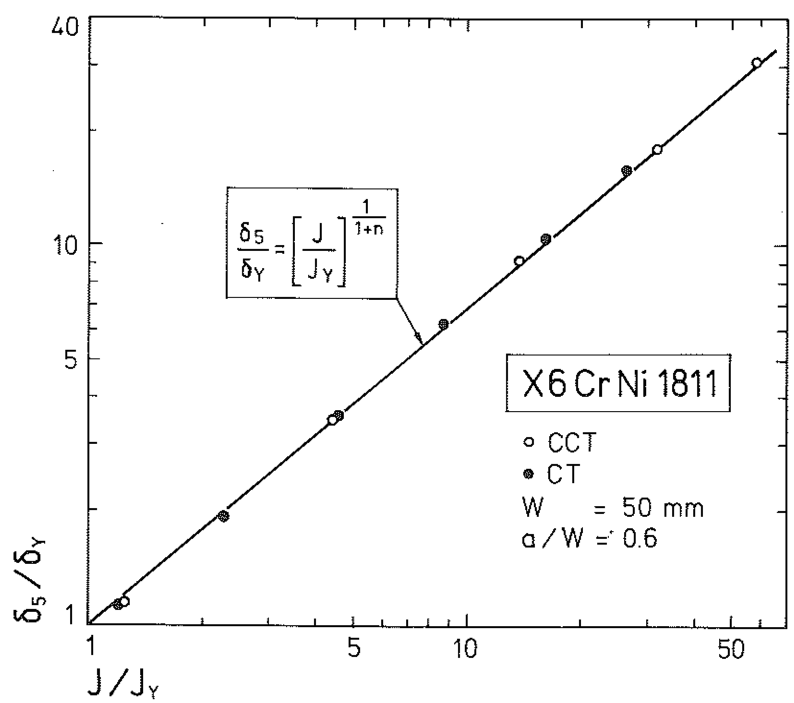

Fig 8 Relationship between δ_5 and J in the fully plastic regime, finite element results from reference (18) compared with ETM

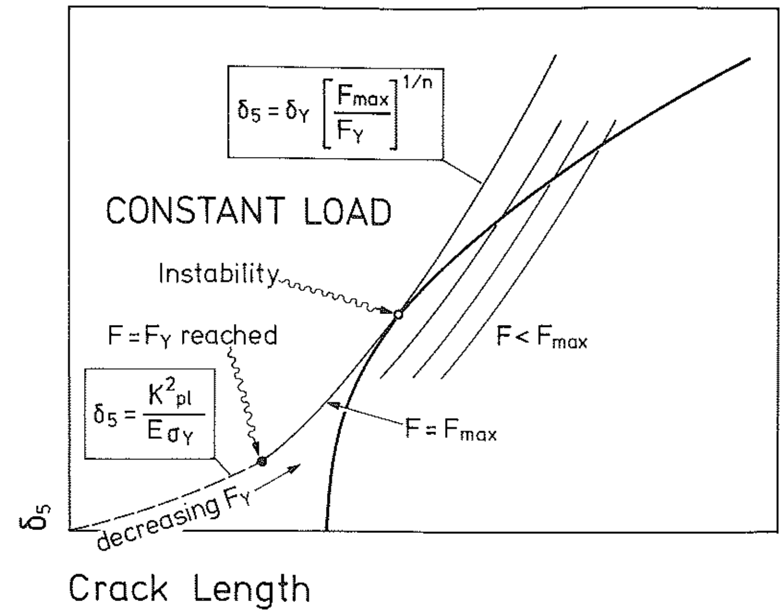

Fig 9 Determination of instability for constant load control using the δ_5–R curve method

for the CT specimen of the previous diagram including data from the regime $F < F_Y$.

Considering equations (2)(7)(8), J can be related to the applied strain, ε, and to δ_5 through

$$\frac{J}{J_Y} = \left(\frac{F}{F_Y}\right)^{(1+n)/n} = \left(\frac{\varepsilon}{\varepsilon_Y}\right)^{1+n} = \left[\frac{\delta_5}{\delta_Y}\right]^{1+n} \tag{20}$$

The right hand side of equation (20) is validated in Fig. 8 which shows a very good agreement between finite element calculations and the predicted relationship between J and δ_5. Thus, equation (20) provides a link between J integral, CTOD, applied load, and applied strain.

Prediction of failure

It is immediately obvious from equation (20) that critical values of applied strain or applied load can be predicted if critical values of J or δ_5 are available as material fracture parameters. Hence, the critical load is given by

$$F_c = F_Y\left(\frac{J_c}{J_Y}\right)^{n/(1+n)} \tag{21}$$

or by

$$F_c = F_Y\left(\frac{\delta_c}{\delta_Y}\right)^{n} \tag{22}$$

and the critical strain is given by

$$\varepsilon_c = \varepsilon_Y \left(\frac{J_c}{J_Y}\right)^{1/(1+n)} \tag{23}$$

or by

$$\varepsilon_c = \varepsilon_Y \frac{\delta_c}{\delta_Y} \tag{24}$$

It should be noted that these expressions contain no safety factors, i.e., they are aimed at predicting rather than assessing on the safe side, with all possible risks in situations which are less clear than simple laboratory specimens. Examples of predicted failure loads have already been reported in references **(11)(12)**. Their degree of accuracy is, of course, similar to that which can be deduced from the diagrams shown in this paper.

In addition to failure predictions using single-valued material parameters, a complete R curve analysis can be done. This is shown schematically in Fig. 9 for load controlled situations. The driving force curve starts in the small scale yielding regime, and after some crack growth the yield condition ($F = F_Y$) in the decreasing net section is reached. It is there where the expressions for full plasticity start to apply. Worked examples have also been reported in references **(11)(12)**.

Discussion and conclusions

The ETM provides simple analytical expressions for predicting the CTOD or the J integral as driving force parameters, in the contained yielding and in the fully plastic regimes. Due to the normalisation of J or CTOD and of load or strain by their respective yield values, geometry independent formulations are obtained which may be regarded as material dependent 'master curves' in contrast to the well known 'design curves' which have no material dependence. The material dependence in the ETM is caused by the strain hardening exponent, see for example equation (20). The closed form solutions ease the application; for example, sensitivity analyses can be easily done. According to the model, the size and geometry independence of the ETM master curve – be it expressed in terms of J or CTOD – means that it is applicable to any structure, provided that the appropriate input information (K solution, F_Y solution, σ_Y, and n) is available. This makes it unnecessary to pre-determine influence functions like in the EPRI Handbook. Thus, an effective tool for predicting driving force parameters is provided by ETM.

Extensive validation of ETM has been performed so far on laboratory specimens made of a variety of materials tested at GKSS and on some finite element calculations carried out at the Technische Hochschule Darmstadt. As the ETM formalisms were derived under the assumption of plane stress conditions the validation refers to experiments and finite element calculations on thin sections. The examples shown in this paper and those compiled in the references give confidence that the ETM, in spite of the simplified underlying mechanical model, is not only easy to apply, as stated in the previous paragraph, but that at least the behaviour of laboratory specimen type geometries can be very well modelled in the regime of plane stress.

The first application of the ETM to a thin-walled large scale structural part shows that the ETM works equally well beyond laboratory scale specimens, as it is demonstrated in a companion paper in this volume **(19)**.

It is expected that the application of the ETM to thick sections and to welded joints will cause problems. Future work will therefore concentrate on these topics.

- Thick sections will be investigated both experimentally and by three-dimensional finite element calculations. The ETM formalisms will then be checked with respect to their ability to model situations outside the regime of plane stress.
- Model welds with longitudinal cracks in the weld metal and in the fusion line are being prepared and are intended to be deformed transverse to the weld, with measurements of δ_5. The ETM has to be modified such that the different stress–strain curves of the base material and of the weld metal are taken into account, when δ_5 is expressed in terms of applied load or applied strain.

It is believed that the future work will enable ETM to provide an easy to use, yet accurate method for assessing the significance of crack-like defects in structural parts.

Acknowledgements

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K.-H. Schwalbe and L. Grüter†*

Application of the Engineering Treatment Model (ETM) to the Prediction of the Behaviour of a Circumferentially Cracked Pipe

REFERENCE Schwalbe, K.-H. and Grüter, L., **Application of the Engineering Treatment Model (ETM) to the prediction of the behaviour of a circumferentially cracked pipe**, *Defect Assessment in Components – Fundamentals and Applications*, ESIS/EGF9 (Edited by J. G. Blauel and K.-H. Schwalbe) 1991, Mechanical Engineering Publications, London, pp. 1125–1133.

ABSTRACT Two pipes out of a series of six tests performed at room temperature were selected for a first application of the Engineering Treatment Model (ETM) to a large structural component. In the pipe tests, crack growth, the crack tip opening displacement, the applied load, and some other quantities, were measured. Calculations were performed to derive best estimates of the maximum applied moment and the amount of crack growth at that point. The results are in good agreement with the experimental findings.

Notation

a	Crack length
a_{eff}	Plasticity corrected crack length
E	Young's modulus
F	Applied force
F_Y	F at incipient net section yielding
K	Linear elastic stress intensity factor
K_{eff}	Plasticity corrected K
$\bar{K}$	Average of K and K_{eff}
M	Applied moment
M_Y	M at incipient net section yielding
n	Strain hardening exponent
t	Wall thickness
Y	Calibration function for K
δ_5	Crack tip opening displacement at fatigue pre-crack tip
δ_Y	δ_5 at incipient net section yielding
ε	Strain
σ	Applied stress
σ_Y	Yield stress
Φ	Crack angle

Introduction

The necessity for making quick assessments of the severity of a crack-like flaw in a structure led to the development of a number of failure assessment

* GKSS-Forschungszentrum, D-2054 Geesthacht, FRG.
† Interatom, D-5060-Bergisch-Gladbach, FRG.