THE C-S CRITERION FOR METALLIC STRUCTURES UNDER MULTIAXIAL HIGH-CYCLE FATIGUE

Andrea Carpinteri
Dept. of Civil-Environmental Engineering and Architecture, University of Parma,
Parco Area delle Scienze 181/A - 43124 Parma - Italy - andrea.carpinteri@unipr.it

Introduction

Actual time-varying loadings are often multiaxial in nature, but the life of metallic structures is generally estimated through uniaxial fatigue test parameters since multiaxial experimental tests are expensive [1-4]. Several fatigue criteria reduce a given multiaxial stress state to an equivalent uniaxial stress condition.

For high-cycle fatigue regime, some criteria proposed in the past decades employ the critical plane concept, that is, fatigue failure assessment is carried out in the plane where amplitude or value of some stress components or a combination of them exhibits a maximum [5-8]. As is well-known, the principal stress directions significantly influence the fatigue fracture plane position and, therefore, some Authors have connected the critical plane orientation with that of the principal stress directions [9, 10]. Since such directions under fatigue loading are often function of time, the critical plane orientation should be determined by relying on averaged principal stress directions (defined through suitable weight functions [11-14]).

The multiaxial high-cycle fatigue criterion known as the C-S (Carpinteri-Spagnoli) critical plane criterion [15], initially proposed for smooth metallic structures under constant-amplitude cyclic loading, has been then modified for examining different cases. First of all, a simplified weighting procedure to determine averaged principal stress axes has been adopted, and the non-zero normal mean stress effect on fatigue limit has been taken into account [16, 17]. Moreover, extensions of the C-S criterion to notched metallic structures subjected to constant-amplitude cyclic loading (by applying the critical point method) [18] and to smooth components subjected to random loading [19] have been published. A new formulation of the criterion for smooth structural components under random loading, based on a frequency-domain approach and on a spectral fatigue damage law, is in progress.

A comparison is made hereafter between some experimental data [20] and theoretical results evaluated by applying both the original C-S criterion and the modified one. Additional comparisons are going to be published [21].

Critical Plane Determination

As was proposed by Brown and Miller [22], the evolution of a fatigue crack can be separated in two stages: (i) Stage 1, when a crack initiates (usually on the external surface of a structural component) in a shear slip plane (Mode II, fatigue crack initiation plane); (ii) Stage 2, when the crack grows in a plane perpendicular to the maximum principal stress direction (Mode I, final fatigue fracture plane).

The procedure to evaluate the critical plane orientation considers both mechanisms (Stage 1 and Stage 2). The fatigue fracture plane orientation is linked with the averaged principal stress directions determined by employing appropriate weight functions (see Refs [11, 15] for the original C-S criterion, and Refs [16, 17] for the modified C-S criterion). Then, the critical plane is correlated to the fatigue fracture plane through an off-angle δ (Ref. [15]) formed by the averaged principal stress direction $\hat{1}$ with the normal **w** to the critical plane (where **w** belongs to the principal plane $\hat{1}\hat{3}$ in Fig. 1a).

The stress vector $\mathbf{S_w}$, the normal stress vector **N** and the shear stress vector **C** acting on the critical plane can be expressed as follows (Fig. 1b):

$$\mathbf{S_w} = \boldsymbol{\sigma} \cdot \mathbf{w} \qquad \mathbf{N} = (\mathbf{w} \cdot \mathbf{S_w})\mathbf{w} \qquad \mathbf{C} = \mathbf{S_w} - \mathbf{N} \tag{11}$$

In the case of multiaxial constant-amplitude cyclic loading, the direction of the normal stress vector $\mathbf{N}(t)$ is time-invariant and, hence, its mean value N_m and amplitude N_a can be easily worked out. On the other hand, the mean value C_m and amplitude C_a of the shear stress vector $\mathbf{C}(t)$ are not uniquely defined due to the time-varying direction of $\mathbf{C}(t)$, which draws a closed path during a loading cycle. The procedure by Papadopoulos [23] can be adopted to compute C_m and C_a.

Original and Modified Formulations for Smooth Specimens

In the original formulation of the C-S criterion (Ref. [15]), the multiaxial fatigue limit condition is expressed by the following quadratic combination of the maximum normal stress ($N_{max} = N_m + N_a$) and the shear stress amplitude (C_a) acting on the critical plane:

$$\sigma_{eq,a} = \sqrt{N_{\max}^2 + \left(\frac{\sigma_{af,-1}}{\tau_{af,-1}}\right)^2 C_a^2} = \sigma_{af,-1} \qquad (2)$$

where $\sigma_{af,-1}$ and $\tau_{af,-1}$ are the normal stress fatigue limit for fully reversed normal stress and shear stress fatigue limit for fully reversed shear stress, respectively.

Since the effect of a tensile mean normal stress superimposed upon an alternating normal stress strongly decreases the fatigue strength of metals, the multiaxial fatigue limit condition in Eq.(2) has been modified as follows [16, 17]:

$$\sigma_{a,eq} = \sqrt{N_{a,eq}^2 + \left(\frac{\sigma_{af,-1}}{\tau_{af,-1}}\right)^2 C_a^2} = \sigma_{af,-1} \qquad (3)$$

where the parameter $N_{a,eq}$ accounts for the effect of the mean normal stress:

$$N_{a,eq} = N_a + \sigma_{af,-1}\left(\frac{N_m}{\sigma_u}\right) \qquad (4)$$

with σ_u = ultimate tensile strength (the yield stress can be used instead of the ultimate tensile strength in the case of elastic-plastic materials). Equation (4) takes into account the linear relationship (proposed by Goodman [24]) between normal stress amplitude and mean normal stress.

The original C-S criterion (Eq. (2) and weight function in Refs [11, 15]) and the modified one (Eq. (3) and weight function in Refs [16, 17]) are applied to some experimental data of high-cycle fatigue tests related to metallic smooth cylindrical specimens subjected to in-phase or out-of-phase bending and torsion [20]:

$$\sigma_l(t) = \sigma_{l,a}\sin(\omega t) + \sigma_{l,m}$$
$$\tau(t) = \tau_a \sin(\omega t - \beta) + \tau_m \qquad (5)$$

where *l* means longitudinal, and the other stress components are equal to zero. In such experimental tests, β (phase angle) is equal to 0° (in-phase), 30°, 60° or 90° (out-of-phase), and the mean stresses are equal to zero. Different values of the stress amplitude ratio $r = \sigma_{l,a}/\tau_a$ are examined. The mechanical properties of the tested materials are reported in Table 1.

The shear stress amplitude C_a against the maximum normal stress $N_{\max}$ acting on the critical plane is plotted in Fig. 2a to 2d. According to the original C-S criterion, fatigue failure occurs if the points with coordinates ($N_{\max}, C_a$) lie out of the ellipse with semi-axes equal to $\sigma_{af,-1}$ and $\tau_{af,-1}$ (see Eq. (2)). By plotting the shear stress amplitude C_a against the equivalent normal stress amplitude $N_{a,eq}$ acting on the critical plane [17] (Fig. 2a' to 2d'), fatigue failure occurs according to the modified C-S criterion if the points with coordinates ($N_{a,eq}$, C_a) lie out of the ellipse with semi-axes equal to $\sigma_{af,-1}$ and $\tau_{af,-1}$ (see Eq. (3)).

For different materials and loading conditions, Figure 2 shows the correlation between such theoretical ellipses and the test results related to fatigue strength at $N_0 = 10^7$ loading cycles. Note that an experimental data point on the theoretical ellipse means a perfect fatigue strength estimation by applying the above criteria. The quality of the estimation determined by means of such criteria can be evaluated through an error index, $I_\sigma = \left[\frac{\sigma_{a,eq} - \sigma_{af,-1}}{\sigma_{af,-1}}\right] \cdot 100\%$. The minimum and the maximum values of such an error index by applying both the original and the modified criterion are reported in Table 1.

Additional comparisons between experimental data and theoretical results deduced by applying both the original and the modified C-S criterion are going to be published (see Ref.[21]).

References

[1] Carpinteri A. (Editor), *Handbook of Fatigue Crack Propagation in Metallic Structures*, Elsevier Science BV, Amsterdam, 1994.

[2] Carpinteri A., De Freitas M., Spagnoli A. (Editors), *Biaxial/Multiaxial Fatigue and Fracture*, Elsevier Science Ltd, U.K., 2003.

[3] Susmel L., Lazzarin, P., A stress-based method to predict lifetime under multiaxial fatigue loadings. *Fatigue Fract. Engng Mater. Struct.* 26, 2003, pp. 1171-1187.

[4] Susmel L., Tovo R., Estimating fatigue damage under variable amplitude multiaxial fatigue loading. *Fatigue Fract. Engng Mater. Struct.* 34, 2011, pp. 1053-77.

[5] Findley W.N., A theory for the effect of mean stress on fatigue of metals under combined torsion and axial load or bending. *J. Engng Industry,* Trans. ASME 81, 1959, pp.301-6.

[6] Matake T., An explanation on fatigue limit under combined stress. *Bull. JSME* 20, 1977, pp. 257-63.

[7] Garud Y.S., Multiaxial fatigue: a survey of the state of the art. *J. Testing Evaluation* 40, 1981, pp.165-78.

[8] McDiarmid D.L., A general criterion for high cycle multiaxial fatigue failure. *Fatigue Fract. Engng Mater. Struct.* 14, 1991, pp. 429-53.

[9] Ohnami M., Sakane M., Hamada N., Effect of changing principal stress axes on low-cycle fatigue life in various strain wave shapes at elevated temperature. *Multiaxial Fatigue ASTM STP* 853, 1985, pp. 622-34.

[10] Macha E., Simulation investigations of the position of fatigue fracture plane in

materials with biaxial loads. *Mat. Wiss. U. Werkstofftech.* 20, 1989, 132-6; 153-63.

[11] Carpinteri A., Macha E., Brighenti R., Spagnoli A., Expected principal stress directions under multiaxial random loading. Part I: theoretical aspects of the weight function method. *Int. J. Fatigue* 21, 1999, pp. 83-8.

[12] Carpinteri A., Brighenti R., Macha E., Spagnoli A., Expected principal stress directions under multiaxial random loading. Part II: numerical simulation and experimental assessment through the weight function method. *Int. J. Fatigue* 21, 1999, pp. 89-96.

[13] Carpinteri A., Brighenti R., Spagnoli A., A fracture plane approach in multiaxial high-cycle fatigue of metals. *Fatigue Fract. Engng Mater. Struct.* 23, 2000, pp. 355-64.

[14] Carpinteri A., Karolczuk A., Macha E., Vantadori S., Expected position of the fatigue fracture plane by using the weighted mean principal Euler angles. *Int. J. Fracture* 115, 2002, pp. 87-99.

[15] Carpinteri A., Spagnoli A., Multiaxial high-cycle fatigue criterion for hard metals. *Int. J. Fatigue* 23, 2001, pp. 135-45.

[16] Carpinteri A., Spagnoli A., Vantadori S., Multiaxial fatigue life estimation in welded joints using the critical plane approach. *Int. J. Fatigue* 31, 2009, pp. 188-96.

[17] Carpinteri A., Spagnoli A., Vantadori S., Multiaxial fatigue assessment using a simplified critical plane-based criterion. *Int. J. Fatigue* 33, 2011, pp. 969-76.

[18] Carpinteri A., Spagnoli A., Vantadori S., Viappiani D., A multiaxial criterion for notch high-cycle fatigue using a critical-point method. *Engng Fract. Mech.* 75, 2008, pp. 1864-74.

[19] Carpinteri A., Spagnoli A., Vantadori S., A multiaxial fatigue criterion for random loading. *Fatigue Fract. Engng Mater. Struct.* 26, 2003, 515-22.

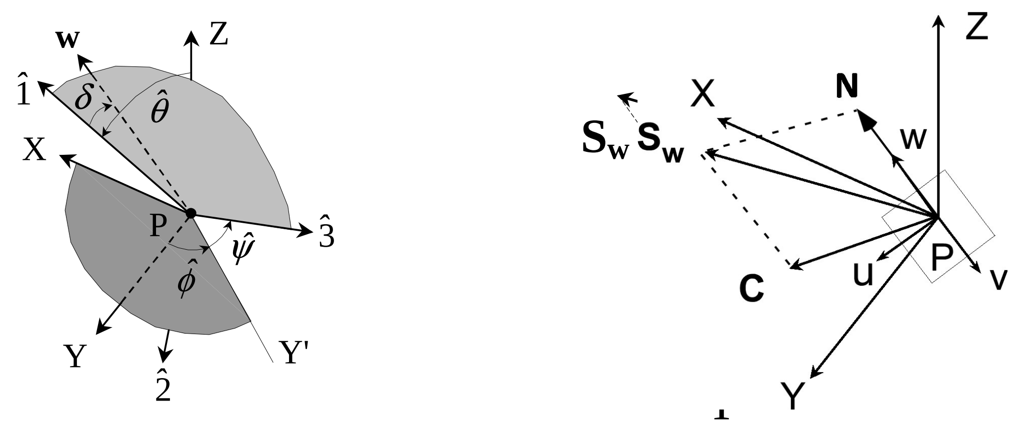
[20] Nishihara T., Kawamoto M., The strength of metals under combined alternating bending and torsion with phase difference. *Memories of the College of Engineering Kyoto Imperial University* 11, 1945, pp. 85-112.

[21] Carpinteri A., Spagnoli A., Vantadori S., Bagni C., Structural integrity assessment of metallic components under multiaxial fatigue: the C-S criterion and its evolution. *Fatigue Fract. Engng Mater. Struct.* 36, 2013, in press.

[22] Brown M.W., Miller K.J., A theory for fatigue failure under multiaxial stress–strain condition. In: *Proceedings of the Institute of Mechanical Engineers* 187, 1973, pp. 745-55.

[23] Papadopoulos I.V., Critical plane approaches in high-cycle fatigue: on the definition of the amplitude and mean value of the shear stress acting on the critical plane. *Fatigue Fract. Engng Mater. Struct.* 21, 1998, pp. 269-85.

[24] Goodman J., *Mechanics Applied to Engineering*. London, Longmans Green, 1899.

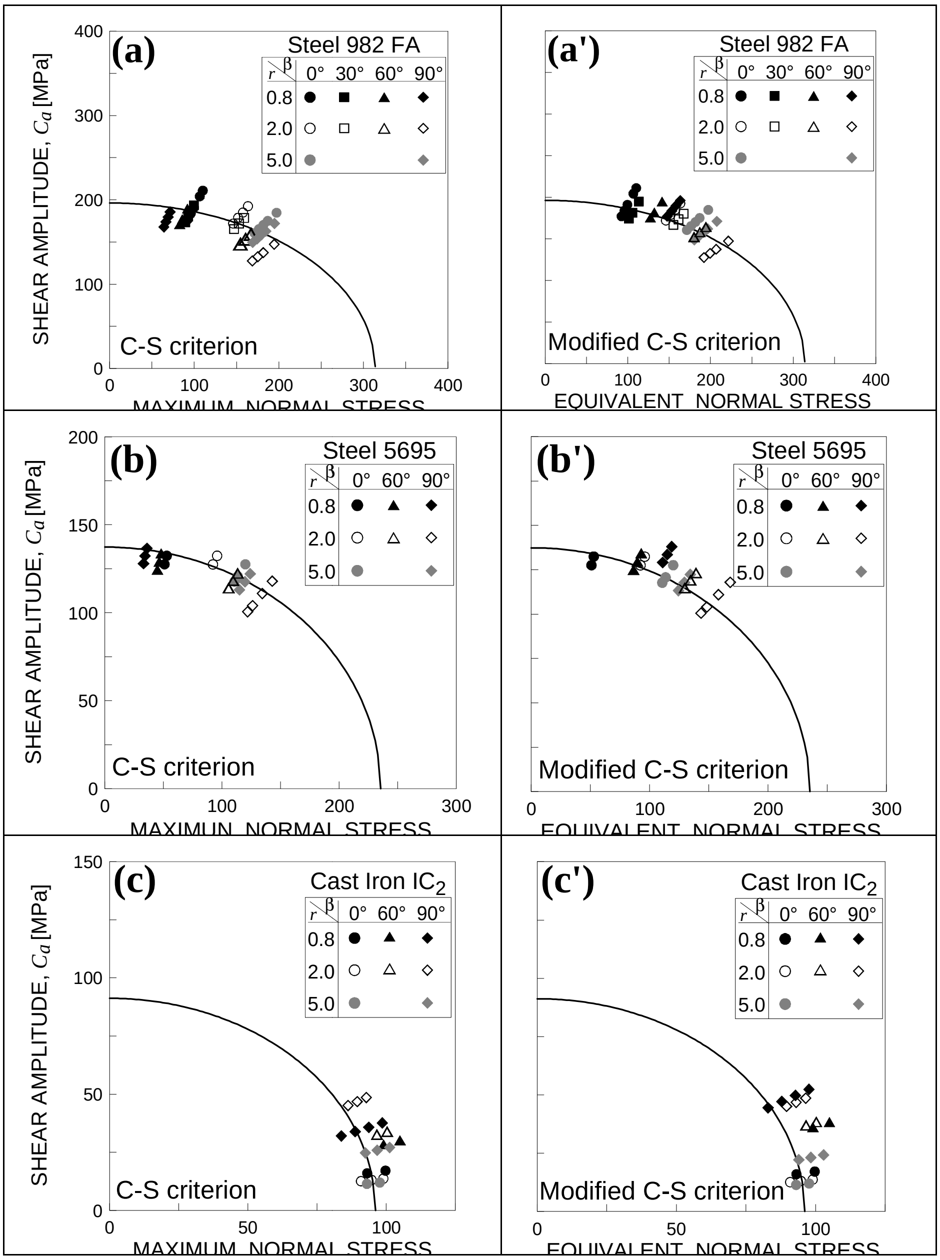


(a) (b)

Figure 1 (a) Correlation between averaged principal stress axes and normal $\mathbf{w}$ to the critical plane; (b) Stress components acting on the critical plane and {Puvw} local coordinate system (*u* and *v* belong to the critical plane).

Table 1 Static, fatigue properties for each examined material [20], and extreme values of the error index I obtained by applying the original and the modified C-S criterion.

Material	σ_u (MPa)	$\sigma_{af,-1}$ (MPa)	$\tau_{af,-1}$ (MPa)	$\frac{\tau_{af,-1}}{\sigma_{af,-1}}$	I_σ range (Original c.)	I_σ range (Modified c.)
Swedish hard steel 982 FA	681	314	196	0.63	-12 / 13%	-7 / 13%
Mild steel 5695	374	235	137	0.58	-7 / 6%	-1 / 12%
Gray cast iron IC2	181	96	91	0.95	2 / 14%	2 / 17%
Duralumin D-30	433	156	100	0.64	-24 / 31%	-17 / 31%



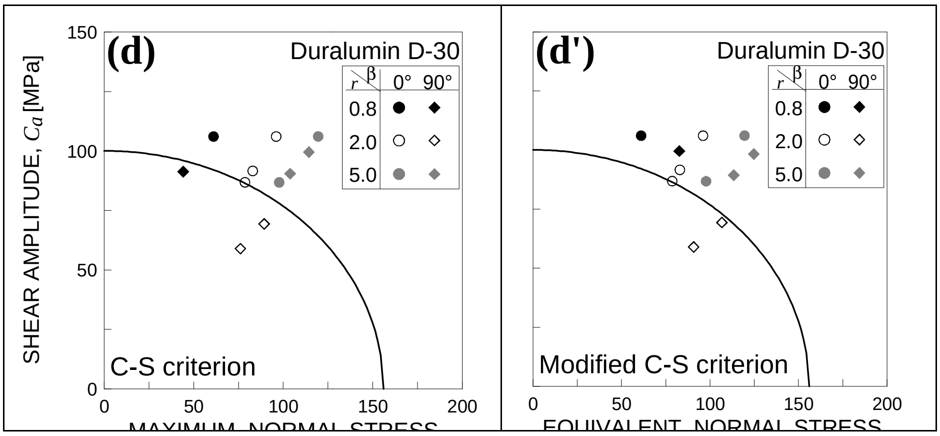


Figure 2 Shear stress amplitude against maximum normal stress (diagrams (a) to (d)) and equivalent normal stress amplitude (diagrams (a') to (d')) acting on the critical plane: theoretical evaluations according to both the original or modified C-S criterion, and experimental results [20].

ORDERED CRACK SYSTEM FORMATION AT THERMAL SHOCK

R.V. Goldstein, N.M. Osipenko

A.Yu. Ishlinsky Institute for Problems in Mechanics,

The Russian Academy of Sciences, Moscow, Russia

(e-mail: goldst@ipmnet.ru)

Introduction

Fracture of brittle materials by cracks formation and growth under thermal action is intensively studied (see, e.g. [1-6]). The problem is interesting, in particular, because of often observed damage and fracture of structural elements fabricated from ceramic materials and glasses at operation under an extremal thermomechanical loading. Multiple fracture is usually observed. A hierarchical ordered crack system (structure of cracks) in the form of a pattern or system of subparallel cracks of different sizes occurs in a loaded body (often on its surface). As a result a body region is unloaded from excessive thermoelastic stresses.

An analysis of thermomechanical fracture is aimed at searching for the conditions of initiation and growth of separate cracks, sequence of events leading to cracks system occurring and growth, conditions of cracks front stability, residual strength and longevity of damaged components, as well as, the methods for determining the thermal strength of materials and components under various loading history. A theoretical analysis of the appropriate problems of solid mechanics for bodies with cracks is rather complex. That is why as a rule the examples of simple stress distributions in the bodies of simple geometry are considered. Simplifying assumptions and approximate methods are used for solving the problems. In particular, this is related to experimental methods which one uses to determine thermostrength properties of materials and components. Majority of the methods are based on evaluation of the thermal stress level by the following semi-emperical formula

$$\sigma = \frac{\alpha E \Delta T K_{\phi}}{(1-\mu)} \qquad (1)$$

where α is the thermal expansion coefficient, E is the Young modulus, μ is the Poisson ratio, ΔT is the temperature drop between the average body temperature and temperature in the region where the stresses are estimated, K_{ϕ} is the shape coefficient.

In this connection asymptotic methods, in which characteristic features of multiple fracture process are used, seem to be interesting. One of such methods is suggested in the given paper. The method is based on the evident effect inherent to the fracture process accompanied by formation of a system of subparallel cracks of different sizes. Indeed, only largest cracks growth is adjusted by the stress field in the scale of the whole body with these cracks. For other cracks in the hierarchy the following effect takes place: the less are the crack length and the distance between them the less is the scale of local stress field having an influence on their growth. In other words, if an advanced process of multiple fracture occurs in a region damaged by initial cracks then thermomechanical processes in separate strips (beams) separated