SOLUTION OF CRACK PROBLEMS BY THE OPTICAL METHOD OF CAUSTICS

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Summary. The limits of applicability and guidelines for the correct determination of stress intensity factors under opening-mode loading conditions by the optical method of caustics are presented. The use of optically anisotropic materials is introduced to obtain a double caustic which provides the state of stress, being plane strain, plane stress or three-dimensional, and therefore, the proper values of stress-optical constants for the correct determination of stress intensity factors.

INTRODUCTION

The optical method of caustics has extensively been used for the determination of stress intensity factors in crack problems (Gdoutos, 2005, Theocaris, Gdoutos, 1972). The method is based on the assumption that the state of stress near the crack tip is plane stress. However, experimental and analytical solutions have shown that the state of stress changes from plane strain near the crack tip to plane stress away from the tip through an intermediate region where the stress state is three-dimensional (Konsta-Gdoutos, Gdoutos, 1992). The changing state of stress results to changing values of stress-optical constants which enter in the equations for the determination of stress intensity factors. For the correct determination of stress intensity factors the proper values of stress-optical constants should be used.

THE OPTICAL METHOD OF CAUSTICS

In the optical method of caustics a specimen is illuminated by a light beam and the reflected or transmitted rays from the front or rear face of the specimen undergo a change of their optical path, due to the variation of the thickness and/or the refractive index dictated by the stress field (Fig. 1). At stress gradients resulting at crack tips, the reflected or transmitted rays generate a highly illuminated three-dimensional surface in space. When this surface is intersected by a reference screen, a bright curve, the so-called caustic curve, is formed. For transparent materials three caustics are formed by the light rays reflected from the front and rear surfaces and those transmitted through the specimen. For opaque materials, only one caustic is formed by the reflected light rays from the front surface of the specimen. The dimensions of the caustic are related to the state of stress near the crack tip. For the case of a mode-I through-the-thickness crack the stress intensity factor K_{exp} is given by

$$K_{\text{exp}} = 0.0934 \frac{D^{5/2}}{z_0\, c\, t\, m^{3/2}} \quad (1)$$

where z_0 is the distance between the specimen and the viewing screen where the caustic is formed, c is the stress optical constant of the specimen under conditions of plane stress, t is the specimen thickness, m is the magnification factor of the optical arrangement defined as the ratio of a length on the reference screen where the caustic is formed divided by the corresponding length on the specimen and D is the transverse diameter of the caustic at the crack tip. The above equation is valid when the state of stress in the vicinity of the crack tip is plane stress, so that the value of stress-optical constant under conditions of plane stress is used.

For optically isotropic materials, the caustic is created by the light rays reflected from the circumference of a circle, the so-called initial curve, which surrounds the crack tip. The radius of the initial curve is given by

$$r = 0.316 D \quad (2)$$

EXPERIMENTAL RESULTS

Experiments have shown that the stress intensity factors determined by caustics corresponding to various values of the radius of the initial curve vary with the distance from the crack tip and reach a plateau value for distances from the tip approximately greater than half the specimen thickness. In these experiments the value of the stress-optical constant entering in the determination of stress intensity factor was obtained equal to the plane stress value. Conditions of plane stress dominate at distances from the tip greater than half the specimen thickness. For distances smaller than half the specimen thickness, where the state of stress is plane strain or three-dimensional the appropriate value of stress-optical constant should be used.

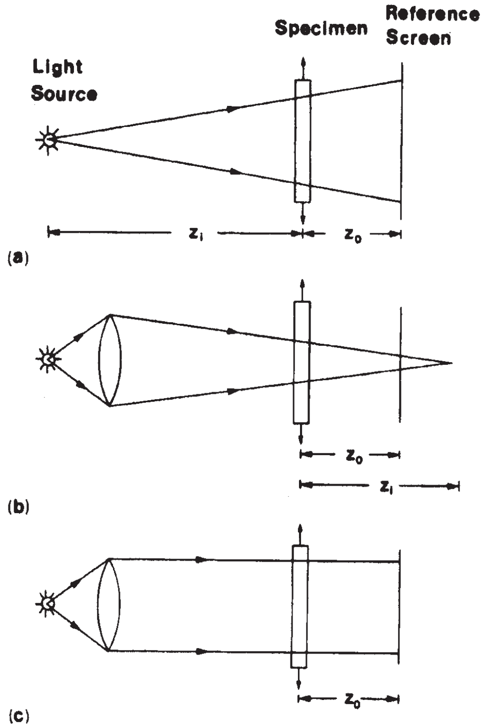

Fig. 1 Optical arrangement for divergent (a), convergent (b) and parallel (c) light

Fig. 2 presents the variation of K_{exp}/K_{th} versus r/d for a value of the specimen thickness $d = 4.5$, and different values of specimen width. Points in figure correspond to different values of the applied load, P, the magnification factor of the optical arrangement, m, the distance between the specimen and the viewing screen where the caustic is formed, z_0, and the specimen thickness, d. Note form figure that the ratio K_{exp}/K_{th} increases with r/d and reaches a plateau value equal to one as the radius of the initial curve takes a limiting value r_c. At that value of $r = r_c$ the state of stress in the neighborhood of the crack tip becomes plane stress. For distances r smaller than r_c the state of stress is three-dimensional, while for values of r larger than r_c plane stress conditions dominate. It was obtained that the critical value of r for which the state of stress becomes plane stress depends not only on d, but also on the geometrical characteristics of the cracked plate, especially the ratio of the crack length to specimen thickness.

LIMITS OF APPLICABILITY OF THE METHOD OF CAUSTICS

The condition that the initial curve of the caustic should lie at distances from the tip approximately greater than half the specimen thickness introduces limitations in the parameters (distance between the specimen and the viewing screen where the caustics is formed, the magnification factor of the optical arrangement, the specimen dimensions and thickness, and applied loads) entering in the determination of stress intensity factors. These factors should be properly selected so that the initial curve lies in the region where plane stress conditions dominate. In that case the value of stress-optical constant corresponding to plane stress should to be used.

In order to obtain caustics generated from the region of plane stress the radius of the initial curve of the caustic should be larger than a fraction of the specimen thickness. By taking this distance equal to half the specimen thickness we obtain

$$\left(\frac{3.385 z_0 c K}{m}\right)^{2/3} > d \qquad (3)$$

Inequality (3) establishes a condition the quantities, z_0, c, K, m, d should satisfy in order to obtain caustics generated by an initial curve that lies in the plane stress region Fig. 3 presents the variation of the critical (maximum) value of specimen thickness, d_c, versus K_I for $z_i = 0.8$ m for a divergent light beam illuminating a notched Plexiglas specimen. z_i represents the distance between the point light source and the specimen. Observe that the critical thickness d_c increases as K_I, z_0, z_i increase.

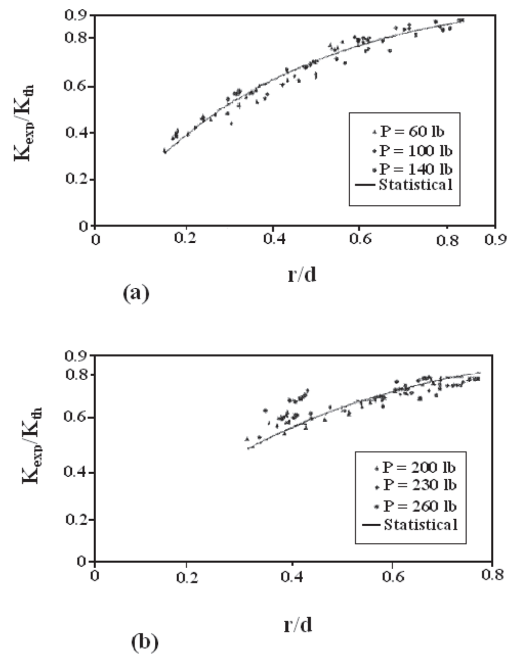

Fig. 2 Variation of K_{exp}/K_{th} versus r/d for a = 15.5 mm, d = 4.5 mm and w = 47.5 mm (a) and w = 63.5 mm (b)

DETERMINATION OF STRESS INTENSITY FACTORS

When the initial curve of the caustic lies at distances where three-dimensional effects dominate the proper value of the stress-optical constant, c, should be used. The value of the stress-optical constant changes from its plane strain value near the tip to its plane stress value at distances away from the tip approximately equal to half the specimen thickness. In order to characterize the three-dimensionality of the stress field near the crack tip an empirical triaxiality factor k is introduced, such that

$$\sigma_z = k\nu(\sigma_x + \sigma_y) \qquad (4)$$

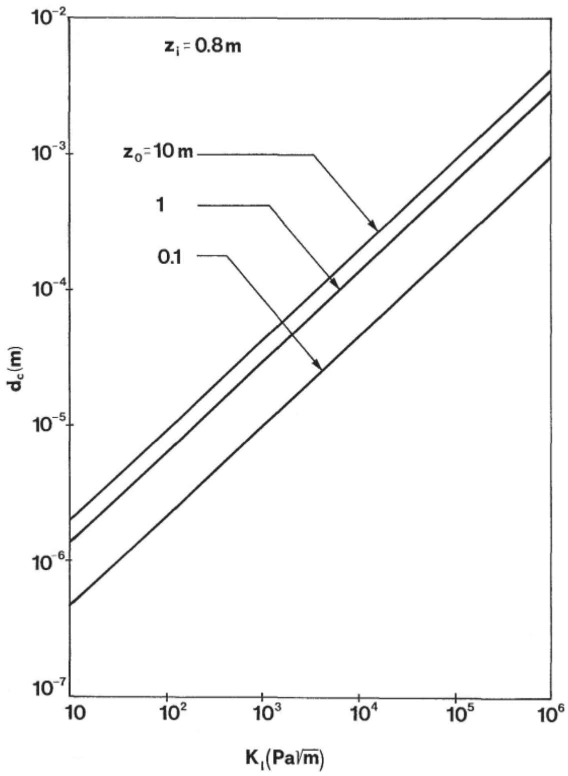

Fig. 3 Variation of maximum value of d (d_c) versus K_I for divergent light. z_i= 0.8 m, and z_0 = 0.1, 1 and 10 m.

where σ_z is the normal stress perpendicular to the plane of the specimen, and σ_x and σ_y are the in-plane stresses. k takes the values of 0 and 1 for plane stress (σ_z = 0) and plane strain [$\sigma_z = k\nu(\sigma_x + \sigma_y)$], respectively.

When the triaxiality factor is determined the corresponding value of the stress-optical constant, c, is calculated which subsequently is used for the determination of stress intensity factor. Fig. 4 presents the variation of the stress-optical constant c_t for transmitted light for Plexiglas (PMMA) versus the triaxiality coefficient k from its plane stress (k = 0) to its plane strain value (k = 1) for various values of the index of refraction n_0 of the surrounding medium. Note that c_t varies linearly with k. From Fig. 4 it is observed that c_t remains almost constant for n_0 = 1.3. This means that when the index of refraction of the medium surrounding the specimen is equal to n_0 = 1.3 the stress-optical constant c_t is independent of the state of stress near the crack tip. Under such circumstances Eq. (1) can be used for the correct determination of stress intensity factor K_I for any values of the parameters entering in Eq. (1).

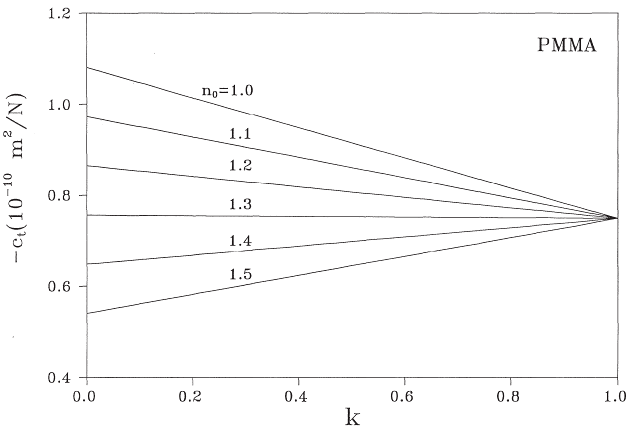

Fig. 4 Variation of stress-optical constant c_t versus triaxiality coefficient k for PMMA for various values of the index of refraction n_0 of the surrounding medium. k = 0 and 1 correspond to conditions of plane stress and plane strain, respectively.

USE OF OPTICALLY ANISOTROPIC MATERIALS

In optically anisotropic materials the variation of the optical path of a light ray traversing the specimen along the two principal stress directions is given by (Theocaris and Papadopoulos, 1981):

$$\Delta s_{t1,2} = c_t\left[(\sigma_1 + \sigma_2) \pm \xi_{r,t}(\sigma_1 - \sigma_2)\right]d \qquad (5)$$

where the coefficient $\xi_{r,t}$ characterizes the optical anisotropy of the material for light rays reflected from the rear face or traversing the specimen. The plus and minus signs in equation correspond to the values σ_1 and σ_2 of the principal stresses. Under such conditions the parametric equations of the caustic are given by:

$$X_{r,t} = \left(\frac{3}{2}c_{r,t}\right)^{2/5}\left[A^{2/5}\cos\theta + \frac{2}{3}A^{-3/5} \right.$$
$$\left. x\left\{\cos 3\theta/2 \pm \frac{3}{4}\xi_{r,t}\sin 2\theta\right\}\right] \qquad (6a)$$

$$Y_{r,t} = \left(\frac{3}{2}c_{r,t}\right)^{2/5}\left[A^{2/5}\sin\theta + \frac{2}{3}A^{-3/5} \times\left\{\sin 3\theta/2 \pm \frac{1}{4}\xi_{r,t}\left(1+3\cos 2\theta\right)\right\}\right] \quad (6b)$$

where:

$$A = \pm\xi_{r,t}\sin\theta + \left[1 \pm \xi_{r,t}\left(7\sin\theta/2 - \sin 3\theta/2\right) + \frac{1}{32}\xi_{r,t}^{2}\left(25+9\cos 2\theta\right)\right] \quad (7)$$

$$C_{r,t} = \frac{\varepsilon z_0 d c_{r,t} K_1}{(2\pi)^{1/2}}$$

The equation of the initial curve is given by:

$$r = r_0 = \left\{\frac{3}{2}C_{r,t}A\right\}^{2/5} \quad (8)$$

Equations (6) express the equations of the caustic curve for optically anisotropic materials. Two caustics are obtained corresponding to the plus and minus signs in equations. These caustics are referred to the two principal stress directions. Note that for $\xi_{r,t} = 0$ equations (6) and (8) reduce to the equations of the caustic for optically isotropic materials [2]. Fig. 5 shows the initial curves and respective caustics in a plate with crack subjected to tension made of birefringent materials with ξ = 0, 0.2, 0.4, 0.6, 0.8 and 1.0. Observe that as ξ increases the shapes of the initial curves and caustics are progressively distorted. The distance between the two caustics increases as ξ also increases. The value of ξ depends on the state of stress, being plane strain, plane stress or three-dimensional. Thus the experimental caustics obtained can be used for the determination of the triaxiality factor *k* and the subsequent calculation of the stress-optical constant for the correct determination of stress intensity factors.

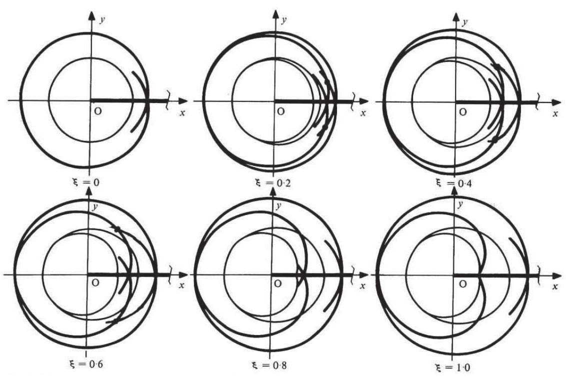

Fig. 5 Initial curves and respective caustics in a plate with crack subjected to tension made of birefringent materials with ξ = 0, 0.2, 0.4, 0.6, 0.8 and 1.0

CONCLUSIONS

From the results of the present work the following conclusions may be drawn:

a. Direct application of the method of caustics without taking special precautions for the determination of stress intensity factors may lead to erroneous results.
b. The material, dimensions of the specimen, applied loads and geometrical dimensions of the optical arrangement should be properly selected to ensure that the initial curve lies in the plane stress region.
c. The above condition is satisfied for high applied loads, small specimen thicknesses, large distances between the specimen and the viewing screen and small magnification factors of the optical arrangement.
d. For specimens made of Plexiglas the stress-optical constant for transmitted light rays is independent of the state of stress around the crack tip for a value of the index of refraction of the medium surrounding the specimen approximately equal to 1.35. Under such condition the plane stress stress-optical constant of the material can be used for any location of the initial curve of the caustic.
e. Optically anisotropic materials can effectively be used for the determination of the state of stress around the initial curve of the caustic and the correct determination of stress intensity factors. For such materials the two caustics formed can be used for the determination of the triaxiality coefficient and the subsequent calculation of the corresponding stress-optical constant.

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